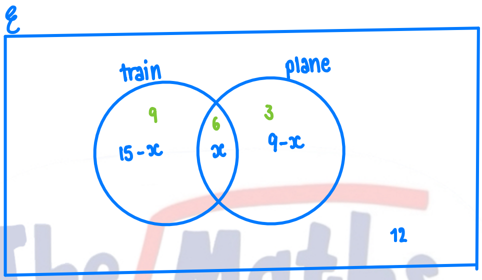
Probability

1. 30 people travelled from London to Manchester for a conference.
Of these people

15 travelled by train
9 travelled by plane
Some travelled by both train and plane
12 did not travel by either train or plane.

Three people are chosen at random from those who travelled by plane.
Find the probability that exactly two of these people also travelled by train.



$$15 - x + x + 9 - x + 12 = 30$$
$$36 - x = 30$$
$$x = 6$$

$$\frac{6}{9} \times \frac{5}{8} \times \frac{3}{7} = \frac{90}{504}$$

$$\frac{90}{504} \times 3 = \frac{15}{28}$$

2. During a week , 500 trains will arrive at a station.
Each train will be late, on time or early.
The probability of a train arriving late is 0.85
The probability of a train arriving on time is 0.07

Calculate an estimate for the number of trains that will arrive early during the week.

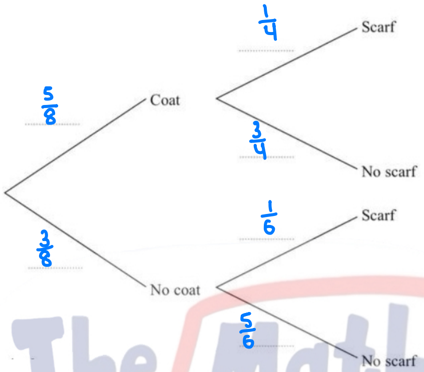
early = 1 - 0.85 - 0.07
= 0 08
= 0.08 × 500
= 40 trains

3. When Joe goes to school in winter, the probability that he wears a coat is $\frac{5}{8}$

If he wears a coat, the probability that he wears a scarf is $\frac{1}{4}$

If he does not wear a coat, the probability that he wears a scarf is $\frac{1}{6}$

(a) Complete the probability tree diagram.



On a day Joe goes to school in winter, calculate the probability that

(b) He is not wearing a coat and is not wearing a scarf.

(c) He is wearing a coat or he is wearing a scarf but he is not wearing both a coat and a scarf.

On a day Joe goes to school in winter, if he is wearing a coat and a scarf then the probability that he is also wearing a hat is $\frac{3}{5}$

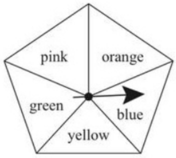
(d) Calculate the probability, that on a day Joe goes to school in winter, he is not wearing all three of a coat, a scarf and a hat.

b) $\frac{3}{8} \times \frac{5}{6} = \frac{5}{16}$

c) $\left(\frac{5}{8} \times \frac{3}{4}\right) + \left(\frac{3}{8} \times \frac{1}{6}\right) = \frac{17}{32}$

d) $1 - \left(\frac{5}{8} \times \frac{1}{4} \times \frac{3}{5}\right) = \frac{29}{32}$

4. Aarya has a biased 5-sided spinner.



When the spinner is spun once, it can land on blue or yellow or green or pink or orange.
Aarya spins the spinner many times and records the colour that the spinner lands on each time. She uses these results to calculate the probability of the spinner landing on each colour.

The table below gives the probability that Aarya calculated for blue, yellow and green.

Colour	blue	yellow	green	pink (x)	orange ($3x$)
Probability	0.22	0.34	0.12	0.08	0.24

Aarya's calculations showed that the probability of the spinner landing on orange is 3 times the probability of the spinner landing on pink.

(a) Complete the table for pink and for orange.

$4x + 0.22 + 0.34 + 0.12 = 1$

$4x = 0.32$

$x = 0.08$

$3x = 0.24$

When Aarya spun the spinner, it landed on green 90 times

(b) Work out an estimate for the total number of times Aarya spun the spinner.

$\frac{90}{0.12} = 750$

5 Arthur has 10 cards .
Each card has a letter on it and the letter spell the word **PICCADILLY**

P I C C A D I L L Y

Arthur selects at random 3 of these cards.
Calculate the probability that he selects P A L in that order.

$\frac{1}{10} \times \frac{1}{9} \times \frac{2}{8} = \frac{1}{360}$

6. Harvey has two bags of counters, bag A and bag B.
Bag A contains 8 red counters, 3 orange counters and 4 yellow counters only.
Bag B contains 3 red counters, 2 orange counters and 5 green counters only.

Harvey takes at random one counter from bag A and one counter from bag B.
(a) Complete the probability tree diagram on the opposite page.

(b) Calculate the probability that the two counters taken have different colours.

Harvey replaces the counters into the bags from which he took the counters.

Chen has a bag, bag C, that contains 10 red counters and x orange counters only.

Chen takes at random one counter from bag A, one counter from bag B and one counter from bag C.

The probability that all three counters that Chen takes have the same colour is $\frac{1}{15}$
(c) Find the value of x.

b) RO + RG + OR + OG + YR + YO + YG

$= \left(\frac{8}{15} \times \frac{2}{10}\right) + \left(\frac{8}{15} \times \frac{5}{10}\right) + \left(\frac{3}{15} \times \frac{3}{10}\right) + \left(\frac{3}{15} \times \frac{5}{10}\right) + \left(\frac{4}{15} \times \frac{3}{10}\right) + \left(\frac{4}{15} \times \frac{2}{10}\right) + \left(\frac{4}{15} \times \frac{5}{10}\right)$

$= \frac{4}{5}$

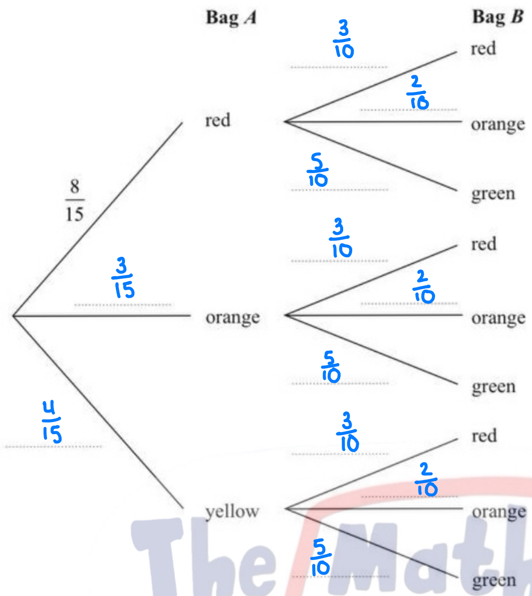
c) $\frac{8}{15} \times \frac{3}{10} \times \frac{10}{x+10} + \frac{3}{15} \times \frac{2}{10} \times \frac{x}{x+10} = \frac{1}{15}$

$15(240 + 6x) = 150(x+10)$

$3600 + 90x = 150x + 1500$

$2100 = 60x$

$35 = x$


Bag A
Bag B
red
orange
yellow
red
orange
green
red
orange
green
red
orange
green
8/15
3/15
4/15
3/10
2/10
5/10
3/10
2/10
5/10
3/10
2/10
5/10

7. Jill travels to work either by car or by train.

Each day that Jill travels to work, the probability that she travels to work by train is 0.1

On a day that Jill travels to work by train, the probability that she will be late for work is 0.05
On a day that Jill travels to work by car, the probability that she will be late for work is 0.01

On a randomly chosen day that Jill travels to work, she is late for work.

Find the probability that Jill travelled to work by train on that day.

$0.01 \times 0.05 = 0.005$

$0\ 005 + (0.9 \times 0.01) = 0.0014$

$\frac{0.005}{0.014} = \frac{5}{14}$

8.There are 12 marbles in bag A and 15 marbles in bag B.

In bag A, there are 7 yellow marbles and 5 red marbles.
In bag B, there are 10 yellow marbles and 5 red marbles.

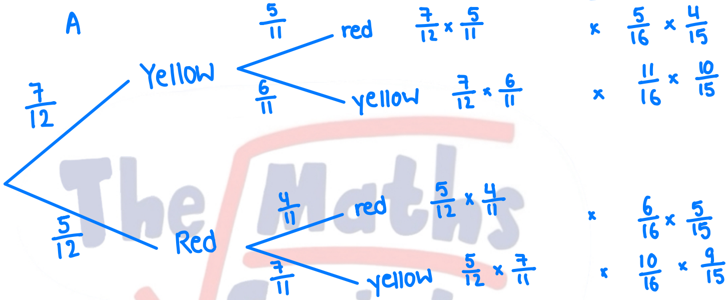
Eugene takes at random **one** marble from bag A and without looking at the marble puts the marble into bag B.

Eugene then takes at random **one** marble from bag A and takes at random **two** marbles from bag B. He places the **three** marbles on a table.

Calculate the probability that the **three** marbles on the table all have the same colour.

A after trsf

B : yellow $= \frac{10}{15}$, red $= \frac{5}{15}$



R from A → B , YYY

$\frac{5}{12} \times \frac{7}{11} \times \frac{10}{16} \times \frac{9}{15} = \frac{35}{352}$

R from A → B , RRR

$\frac{5}{12} \times \frac{4}{11} \times \frac{6}{16} \times \frac{5}{15} = \frac{5}{264}$

Y from A → B , YYY

$\frac{7}{12} \times \frac{6}{11} \times \frac{11}{16} \times \frac{10}{15} = \frac{7}{48}$

Y from A → B , RRR

$\frac{7}{12} \times \frac{5}{11} \times \frac{5}{16} \times \frac{4}{15} = \frac{35}{1584}$

$\frac{35}{352} + \frac{5}{264} + \frac{7}{48} + \frac{35}{1584} = \frac{907}{3168}$

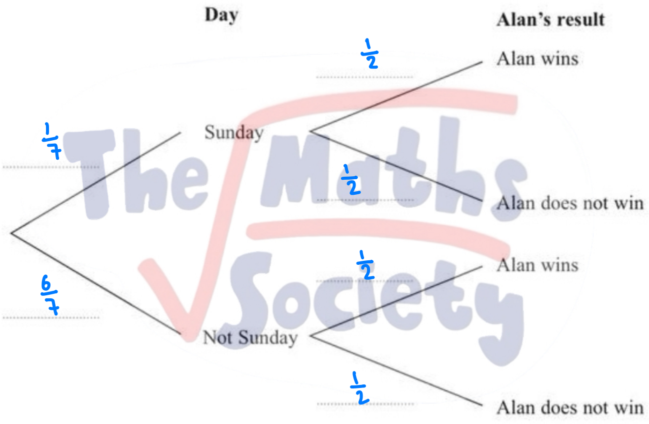
9. Over the course of a week of seven days, Alan plays a series of games.
He plays exactly one game each day.

On Sunday, the probability that Alan wins his game is $\frac{1}{2}$

On a day other than Sunday, the probability that Alan wins his game is $\frac{1}{3}$

The incomplete probability tree diagram below is to give information about Alan's result on a randomly chosen day in a week of seven days.

(a) Complete the probability tree diagram below.



(b) Find the expected number of games Alan will win in a week of seven days.

$$\left(\left(\frac{1}{7}\times\frac{1}{2}\right)+\left(\frac{6}{7}\times\frac{1}{2}\right)\right)\times 7$$

$$= 2.5$$

10. Ramesh, Maya, Kalil, Chen and Andreia each have a bag containing an identical set of six cards.

There is a number on each of the six cards.
Here are the cards in each of the bags.

2	**4**	**4**	**7**	**9**	**10**

Ramesh takes at random **one** of the six cards in his bag.

(a) Write down the probability that the number on the card Ramesh takes is a prime number. $\frac{2}{6} = \frac{1}{3}$

Maya takes at random from her bag **two** of the six cards in her bag.
(b) Find the probability that neither of the two cards has a number 4 on it. $\frac{4}{6} \times \frac{3}{5} = \frac{2}{5}$

Kalil takes at random from his bag **two** of the six cards in his bag
(c) Find the probability that the total of the two numbers on the cards is 11
$2\left(\frac{1}{6} \times \frac{1}{5}\right) + 2\left(\frac{2}{6} \times \frac{1}{5}\right) = \frac{1}{5}$

Chen takes at random **one** card at a time, without replacement, from her bag until she gets a card with a number 4 on it. She then stops taking cards from her bag

(d) Find the probability that Chen stops taking cards from her bag before she takes the fourth card. $\frac{2}{6} + \left(\frac{4}{6} \times \frac{2}{5}\right) + \left(\frac{4}{6} \times \frac{3}{5} \times \frac{2}{4}\right) = \frac{4}{5}$

Andreia puts another card with a number on it into her bag so that she has seven cards in her bag.
The mean of the numbers on the seven cards in Andreia's bag is 8.

(e) Find the value of the number on the card that Andreia put into her bag.

$$\frac{2+4+4+7+9+10+x}{7} = 8$$

$$x = 56 - 36$$

$$= 20$$

11. There are 45 young people in a youth club.
The table below gives information about the heights of these young people.

Height (h cm)	Frequency
$140 < h \leqslant 150$	5
$150 < h \leqslant 155$	8
$155 < h \leqslant 160$	11
$160 < h \leqslant 165$	6
$165 < h \leqslant 170$	12
$170 < h \leqslant 190$	3

(a) Find the class interval that contains the median.

$155 < h \leq 160$

(b) Calculate an estimate of the mean height, in cm to one decimal place, of these young people.

$(145 \times 5) + (152.5 \times 8) + (157.5 \times 11) + (162.5 \times 6) + (167.5 \times 12) + (180 \times 3) \,/\, 45 = 160.1$

One of the young people is selected at random.

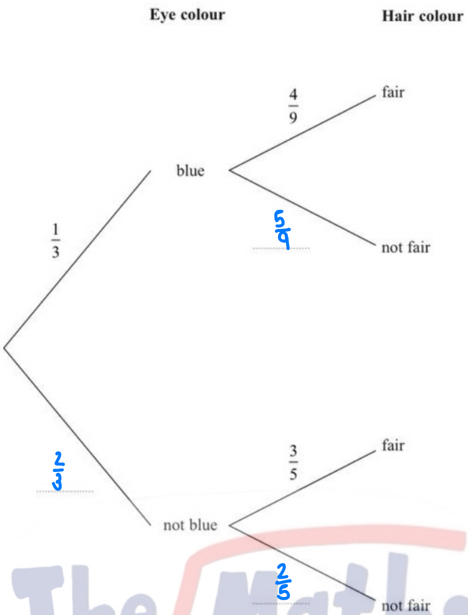
The incomplete probability tree diagram on the opposite page gives information about the eye colour and the hair colour of this young person.

(c) Complete the probability tree diagram.

Given that, for these young people, hair colour is independent of height

(d) calculate the probability that the young person selected has fair hair and a height of more than 160 cm.

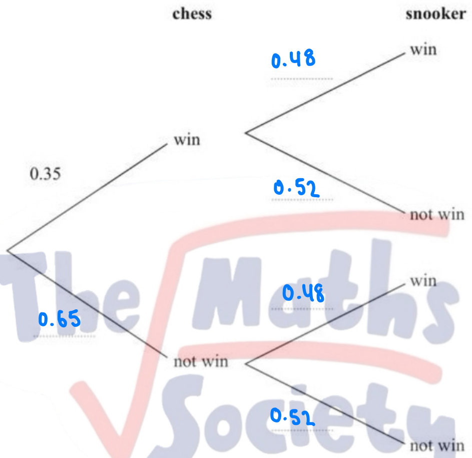
$\left(\frac{21}{45} \times \frac{1}{3} \times \frac{4}{9}\right) + \left(\frac{21}{45} \times \frac{2}{3} \times \frac{3}{5}\right) = \frac{518}{2025}$


Eye colour
Hair colour
fair
4/9
blue
5/9
not fair
1/3
fair
3/5
2/3
not blue
2/5
not fair

12. Neeta is going to play one game of chess and one game of snooker.

The probability that she will win the game of chess is 0.35
The probability that she will win the game of snooker is 0.48
The two events are independent.

(a) Complete the probability tree diagram.



(b) Find the probability that Neeta will win only one of the games.

$(0.35 \times 0.52) + (0.65 \times 0.48) = 0.494$

13. The table gives information about the weights, in grams, of 40 adult mice.

Weight (w grams)	Frequency
$17 \leqslant w < 19$	8
$19 \leqslant w < 21$	3
$21 \leqslant w < 22$	15
$22 \leqslant w < 23$	8
$23 \leqslant w < 25$	6

(a) Calculate an estimate for the mean weight, in grams to 3 significant figures of these adult mice.

$$\text{mean} = \frac{(18\times 8)+(20\times 3)+(21.5\times 15)+(22.5\times 8)+(24\times 6)}{40}$$

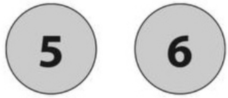
$$= 21.3$$

One of these adult mice is selected at random.

(b) Find the probability that this mouse weighs less than 22 grams.

$$\frac{26}{40} = \frac{13}{20}$$

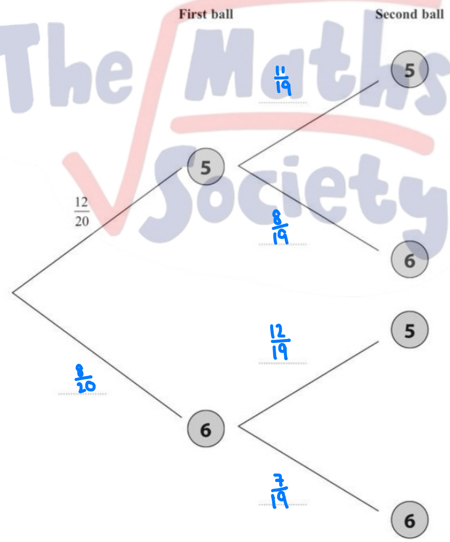
14. A bag contains 20 balls. Each ball has either the number 5 on it or the number 6 on it.



There are 12 balls with the number 5
There are 8 balls with the number 6

Ben takes at random 2 balls from the bag.

(a) Complete the probability tree diagram.



(b) Calculate the probability that the sum of the numbers on the two balls is greater than 10
Give your answer as a fraction.

$$\left(\frac{12}{20} \times \frac{8}{19}\right) + \left(\frac{8}{20} \times \frac{12}{19}\right) + \left(\frac{8}{20} \times \frac{7}{19}\right)$$

$$= \frac{62}{95}$$

15. Peter has two boxes of bricks, box A and box B.
Each box contains only red bricks, green bricks and yellow bricks.

In box A there are 15 bricks of which

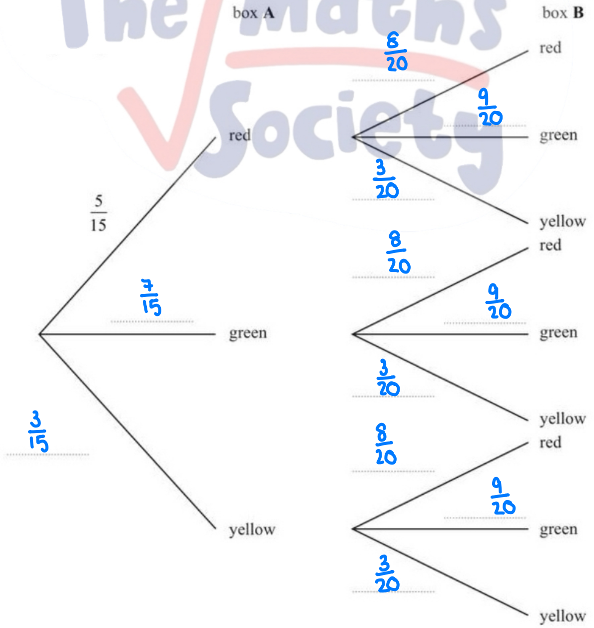
5 are red,
7 are green,
and 3 are yellow.

In box B there are 20 bricks of which

8 are red,
9 are green,
and 3 are yellow.

Ami is going to take a brick at random from box A and a brick at random from box B.
She is going to put these two bricks on a table.

(a) Complete the probability tree diagram.



(b) Calculate the probability that the two bricks on the table are different colours.

Peter has a third box of bricks, box C.
This box also contains only red bricks, green bricks and yellow bricks.

After taking a brick from box A and a brick from box B, Ami then takes at random a brick from box C.
Ami then puts this brick with the other two bricks on the table.

The probability that there are three yellow bricks on the table is $\frac{19}{1000}$

(c) Find the least number of bricks that were in box C before Ami took a brick from box C.

b) $1 - \left[\left(\frac{5}{15} \times \frac{8}{20}\right) + \left(\frac{7}{15} \times \frac{9}{20}\right) + \left(\frac{3}{15} \times \frac{3}{20}\right)\right]$

$= \frac{47}{75}$

c) $0.2 \times 0.15 \times x = 0.019$

$x = \frac{0.019}{0.2 \times 0.15} = \frac{19}{30}$

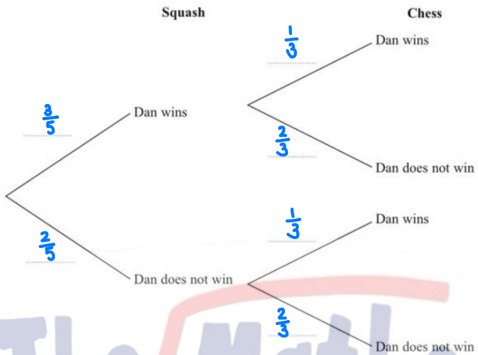
$\therefore$ 30 bricks

16. Dan is going to play one game of squash and one game of chess against his friend.

The probability that Dan will win the game of squash is $\frac{3}{5}$

The probability that Dan will win the game of chess is $\frac{1}{3}$

(a) Complete the probability tree diagram.



(b) Find the probability that Dan will win exactly one game.

Given that Dan won exactly one game,

(c) calculate the probability that he won his game of squash.

b) $\left(\frac{3}{5} \times \frac{2}{3}\right) + \left(\frac{2}{5} \times \frac{1}{3}\right) = \frac{8}{15}$

c) $\frac{\left(\frac{3}{5} \times \frac{2}{3}\right)}{\frac{8}{15}} = \frac{3}{4}$

17. A box contains 8 green counters and 2 white counters only.

Peter takes at random 2 counters from the box.

(a) Calculate the probability that Peter will take 1 green counter and 1 white counter.

A bag contains 28 blue beads and n red beads only.
Naasir selects a bead from the bag at random.

(b) Explain why the probability of the bead being red cannot be $\frac{6}{11}$

Naasir keeps the first bead and selects a second bead at random from the bag.
The probability of both beads being different colours is $\frac{1}{2}$

Given that there are fewer blue beads than red beads,

(c) calculate the probability that both beads are blue.
Show clear algebraic working

a) $\left(\frac{8}{10} \times \frac{2}{9}\right) + \left(\frac{2}{10} \times \frac{8}{9}\right) = \frac{16}{45}$

b) $\frac{n}{n+28} - \frac{6}{11}$

$11n - 6n = 168$

$n = 33.6$

$\therefore$ n is not an integer

c) $\frac{28}{28+n} \times \frac{n}{28+n-1} \times 2 = \frac{1}{2}$

$n^2 - 57n + 756 = 0$

$(n-21)(n-36) = 0$

$n = 21$ (or) $n = 36$

(reject)

$\frac{28}{28+36} \times \frac{27}{28+35} = \frac{3}{16}$

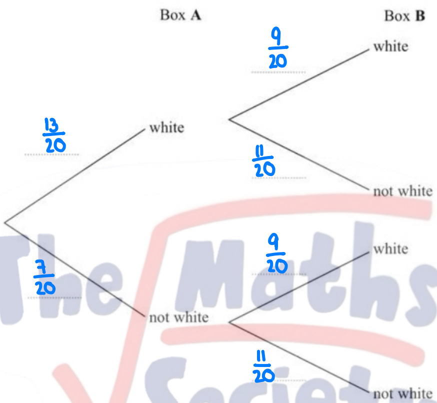
$$\text{Solutions of } ax^2 + bx + c = 0 \text{ are } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

18. Ted has two boxes of buttons, Box A and Box B

There are 20 buttons in Box A and 20 buttons in Box B

There are 13 white buttons in Box A
There are 9 white buttons in Box B

Ted takes at random a button from Box A and a button from Box B
(a) Complete the probability tree diagram.



(b) Calculate the probability that Ted takes at least one white button.

Ted also has a bag of buttons.
There are 30 buttons in the bag.
There are x white buttons in the bag.

Ted takes at random a button from Box A, a button from Box B and a button from the bag.

The probability that of the 3 buttons Ted has, 2 of the buttons are white is $\frac{337}{1000}$

(c) Work out the value of x
Show clear algebraic working.

b) $1 - \left(\frac{7}{20} \times \frac{11}{20}\right) = \frac{323}{400}$

c) $WWN = \left(\frac{13}{20} \times \frac{9}{20} \times \frac{30-x}{30}\right) = \frac{3510-117x}{12\,000}$

$+$

$WNW = \left(\frac{13}{20} \times \frac{11}{20} \times \frac{x}{30}\right) = \frac{143x}{12\,000}$

$+$

$NWW = \left(\frac{7}{20} \times \frac{9}{20} \times \frac{x}{30}\right) = \frac{63x}{12\,000}$

$\frac{89x + 3510}{12000} = \frac{337}{1000}$

$89\,000x + 3510\,000 = 4044\,000$

$89x + 3510 = 4044$

$89x = 534$

$x = 6$